Further investigations:
Look for direct relationships and discuss them with your student. For example, if you earn an hourly wage, how will your earnings change as the number of hours you work changes?

Examine savings accounts with your student. How does the amount of interest earned (on a given principal at a fixed rate) change as time increases?

Review your telephone or cell phone bill with your student. How does the bill change when additional minutes are used?

Look for inverse relationships and discuss them with your student. For example, on a trip, how does travel time change when you change your rate of speed?

On a car trip, choose a distant object such as a telephone pole. Ask your student to notice how the height of the object seems to change as your car gets closer to the object. Discuss the relationship between distance away and apparent height.

Terminology:
Constant of proportionality: A value, \( k \), that does not change; it indicates the relationship between the variables. In a direct proportion, \( k \) is the ratio of the variables. In an inverse proportion, \( k \) is the product of the variables.

Direct variation: A relationship between 2 variables in which one is the constant multiple of the other. \( x \) and \( y \) are directly proportional, if \( y = kx \) where \( k \) denotes a constant of proportionality and \( k \neq 0 \). Direct variation is sometimes called direct proportion.

Inverse variation: A relationship between two variables in which the product is a constant. \( x \) and \( y \) are inversely proportional, if \( xy = k \) where \( k \) denotes a constant. Inverse variation is sometimes called indirect variation or indirect proportion.

Book’em:
Chapter 5 in The Man Who Counted by Malba Tahan

Related Files:
www.ceismc.gatech.edu/csi

Values that Vary

Students will:
- Collect, organize, and graph data that relates two variables
- Draw pictures and use manipulatives to demonstrate a conceptual understanding of proportion
- Solve problems using proportional reasoning
- Recognize and represent direct and inverse proportions in multiple ways
- Determine and interpret the constant of proportionality
- Explain how a change in one variable affects another

Classroom Cases:
1. Jean is traveling and keeping a record of her distance and time as shown.

<table>
<thead>
<tr>
<th>Time (hrs)</th>
<th>0.75</th>
<th>1.5</th>
<th>2</th>
<th>3</th>
<th>3.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distance (miles)</td>
<td>42</td>
<td>84</td>
<td>112</td>
<td>168</td>
<td>196</td>
</tr>
</tbody>
</table>

Plot the data and describe the relationship between distance and time. If the relationship is proportional, determine the constant of proportionality. Write an equation to represent the relationship; use it to predict how far she will go in 8 hours.

Case Closed - Evidence:
Distance and time have a direct proportional relationship. As time increases, distance increases by a multiple of 56. The constant of proportionality is distance/time or 56 miles per hour.

The relationship can be represented by \( D = 56t \) where \( D \) = distance traveled and \( t \) = time traveled. Then \( D = 56 \times 8 = 448 \) miles traveled in 8 hours.

2. Tony just baked 24 cookies which he plans to give to his friends. He may have 1, 2, 3, or more friends. Make a table to show how many cookies each friend will get if the friends share equally. Plot the information in your table and describe your graph. If the relationship between number of friends and number of cookies is proportional, determine the constant of proportionality. Write an equation to represent the relationship and use it to predict how many cookies each person will get if Tony shares with 30 friends.

Case Closed - Evidence:
Number of friends and number of cookies vary inversely. As friends increase, the cookies per friend decrease. The constant of proportionality is 24. \( f \cdot c = 24 \) where \( f = \) no. of friends and \( c = \) no. of cookies per friend. For 30 friends, \( 30c = 24 \) and \( c = 4/5 \) cookie.

3. Taneisha and Marcus are having a party.
   A. Taneisha is making punch. Her recipe calls for 3 cups of pineapple juice for every 5 cups of ginger ale. How many cups of each will she need to make 120 cups of punch?
   B. Marcus is replenishing the punch and notices that as more guests arrive, each guest gets fewer ounces of punch. When six guests are present, one pitcher will supply each guest with 5 ounces of punch. When 10 guests are present, they empty the pitcher with 3 ounces each. How many ounces does the pitcher hold?

Case Closed - Evidence:
The relationship between pineapple juice and punch is a direct proportion as is the relationship between ginger ale and punch. There are 3 cups of juice and 5 cups of ginger ale for every 8 cups of punch. 

\[ \frac{\text{Juice}}{\text{Punch}} = \frac{3}{8} = \frac{15}{120} \]

\[ \frac{\text{ginger ale}}{\text{punch}} = \frac{5}{8} = \frac{15}{120} \]

Taneisha needs 45 cups of juice and 75 cups of ginger ale. The relationship between guests and servings is inverse. Since 6×5oz = 30oz and 10×3oz = 30oz, 30 is the constant of proportionality and the total ounces the pitcher holds.