Further investigations:
Encourage your student to compare payment plans from two cell phone companies. Suggest that she graph the monthly service charge and per-minute fees for each company. Based on how she might use a cell phone, which plan is best for her? Which plan would be best for someone who has no other telephone? Which plan would be best for someone who only uses a cell phone for emergencies? When would the plans have the same charge? When your student is interested in purchasing something, ask him to represent the options mathematically. For example, with $10.00, what combinations of hamburgers and fries can he buy?
With your student, look at graphs that have more than one line. Ask her what is happening when the lines cross each other? What would it mean if they never cross? When would one line be the better choice than the other and why?

Terminology:
Constraint: A restriction placed on variables in a problem situation. Inequalities can represent constraints.
Feasible region: The area on a graph consisting of all the points that satisfy the constraints.
Graphing method: A technique for solving a system of equations that involves reading the coordinates of the point of intersection of the graphs of the equations in the system.
Combination method: A technique for solving a system of equations that involves combining two equations in order to eliminate one of the variables and solving for the remaining variable.
Linear inequality: A mathematical statement which expresses a relationship between two unequal linear expressions. The graph of a linear inequality is a half-plane on one side of a related line.
Substitution Method: A technique for solving a system of equations that involves replacing one variable with an equivalent expression and solving for the remaining variable.
System of linear equations: Two or more equations that together define a relationship between variables usually in a problem situation. A system of equations can have no solution, one solution, or many solutions.
System of linear inequalities: Two or more inequalities that together define a relationship between variables, usually in a problem situation.

Related Files:
www.ceismc.gatech.edu/csi

Systems of Equations and Inequalities

Students will:
• Solve systems of equations geometrically and algebraically
• Determine the number of solutions a system of equations will have before attempting to solve the system
• Translate a problem situation into a system of equations or inequalities
• Interpret the solution to a system of equations in terms of the original problem
• Determine the solution to a system of linear inequalities by graphing

Classroom Cases:
1. E-tunes and Muzic provide music download services. E-tunes charges a $10 annual fee and $2.00 per song downloaded. Muzic has no annual fee, but charges $2.00 per song. Which service would you prefer and why?

<table>
<thead>
<tr>
<th>No. of Songs</th>
<th>E-Tunes Cost</th>
<th>Muzic Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$10.75</td>
<td>$2.00</td>
</tr>
<tr>
<td>2</td>
<td>$11.50</td>
<td>$4.00</td>
</tr>
<tr>
<td>3</td>
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<td>$6.00</td>
</tr>
<tr>
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<td>$18.00</td>
</tr>
<tr>
<td>10</td>
<td>$17.50</td>
<td>$20.00</td>
</tr>
</tbody>
</table>

Case Closed - Evidence:
I would prefer E-tunes because I plan to download at least one song a month and E-tunes is cheaper for 9 or more songs. Both services cost the same for 8 songs.

2. Six hundred tickets were sold for the Spring Concert, and the total income from ticket sales was $2716.50. If adult tickets cost $5.25 and student tickets cost $3.75, how many of each type of ticket were sold?

Case Closed - Evidence:
I will let a = number of adult tickets and s = number of student tickets
Number of tickets: a + s = 600
Value of tickets: 5.25a + 3.75s = 2716.50
From the first equation, s = 600 – a.
Substituting,
5.25a + 3.75(600 – a) = 2716.50
5.25a + 2250 – 3.75a = 2716.50
1.5a = 466.50
a = 311
s = 600 – a = 600 – 311 = 289

3. Coach Woods is buying awards for the end of season banquet. Based on past experience, she knows she will need at least twice as many certificates as trophies, at least 2 trophies, and fewer than 12 awards. If certificates cost $3 and trophies cost $15, what is the most Ms. Woods could spend on awards and what is the least she could spend?

Case Closed - Evidence:
I graphed the situation constraints as shown at right. There are infinitely many values in the feasible region (dark green triangle), but only whole numbers make sense here. The fewest awards Ms Woods can buy is 6 certificates and 2 trophies; this will cost $48. The most awards the coach can buy is 11; the most expensive combination is 2 trophies and 9 certificates for $57