Circles and Spheres

Students will:

- Apply properties of circles to solve problems involving arcs, angles, sectors, chords, tangents and secants
- Calculate and compare surface areas and volumes of spheres
- Justify measurements and relationships in circles using geometric and algebraic properties

Classroom Cases:

1. To make their product seem like a better buy, Charlie’s Chocolates, a candy company, increased the size of their chocolate balls. The new candies increased from a 2cm diameter to a 3cm diameter, without increasing the price. In fact, both candies contain the same amount of chocolate. The 3cm balls are hollow spherical shells, while the 2cm balls are solid chocolate. How thick are the spherical chocolate shells that Charlie’s Chocolates are now selling?

2. Circle B and Circle F are tangent. Circle B has a radius of 2”. \( m\angle BCA = 30^\circ \).

Case Closed – Evidence:

Let \( x \) represent the thickness of the shell as shown in the diagram. Then \( (1.5 - x) \) will be the radius of the hollow center of the new candy.

\[
\text{volume} = \frac{4}{3} \pi (1.5)^3 - \frac{4}{3} \pi (1.5-x)^3
\]

But \( \text{volume} = \frac{4}{3} \pi ((1.5)^3 - (1.5-x)^3) \)

Graphing the equation to the left, I see that there is only one real root between 0 and 1.5, and it is approximately 0.1658. The thickness of the new candy shell will be about 0.1658.

Case Closed – Evidence:

\( m\angle CDA = 60^\circ \)

\( DG \) and \( DE \) are tangent to both circles.

Find the length of \( \overline{AC} \) and \( \overline{EG} \)

Case Closed – Evidence:

\( m\angle CBA = 120^\circ \) (quadrilateral ABCD has a sum of interior angles of 360\(^\circ\).

\( 360-90-90-60 = 120 \)

\( \frac{120}{360} = \frac{1}{3} \) (circumference of circle B)

\[
\text{length of } \overline{AC} = \frac{120}{360} \times 2\pi r = 4.1866
\]

Draw segment from D to F, forming 4 30\(^\circ\)-60\(^\circ\)-90\(^\circ\) triangles.

The hypotenuse of triangle BCD is 4”.

\( DF = (4 + 2 + x) \) also \( DF = 2x \) where \( x \) is the radius of circle F.

\( 6 + x = 2x \) therefore the radius of circle F is 6 inches.

The length of \( EG = \frac{1}{3} (12x) = 12.56 \) inches.

3. An arch over a door is 40 inches wide and 15 inches high. Find the radius of the circle that contains the arch.

Case Closed – Evidence:

Construct circle A with chord \( \overline{BC} = 40^\circ \). The perpendicular bisector of \( \overline{BC} \) is a diameter of circle A. The shortest distance from the midpoint of \( \overline{BC} \) to the circle is 15”.

Let \( x \) be the length of the diameter = 15”.

\( 20(20) = 15x \) (If two chords intersect in a circle, then the products of the measures of the segments of the chords are equal.)

\[
\frac{80}{3} + 15 = \frac{20 + 5}{6} \text{ inches}
\]